Combining long and short duration areal reduction factors

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Abstract

The Bureau of Meteorology (the Bureau) provides new Intensity-Frequency-Duration (IFD) design rainfalls in the form of point values. However, as design flood estimates are often required for large catchments, areal design rainfall estimates are necessary. Therefore areal reduction factors (ARFs), which are the ratios between point and areal design rainfall for a given catchment area, duration and annual exceedance probability (AEP) were produced to compensate for the effect of rainfall from large events rarely being spatially uniform.

In light of the large differences in data availability for durations less than 1 day and those greater than 1 day, it was necessary to derive short and long duration ARFs separately. The Bureau derived long duration ARFs using a modified Bell's method for all of Australia and developed an equation to model these values. WMA Water produced short duration ARFs using a method more suited to the smaller data set.

To maintain consistency in the use of ARFs, an equation was fitted to the short duration values and comparisons were drawn between the long and short duration data. Given that differences were present between modelled values for the 1080min duration, a simple interpolation technique is recommended to obtain ARF values between 720min and 1day.

By investigating the differences between the long and short duration data and developing techniques to use the values together, ARFs have been made available for all durations from 30min to 7days, all catchment areas from 0 to 30,000km2 and all annual exceedance probabilities from 50% to 1%.

1. INTRODUCTION

In 2013 the Bureau of Meteorology (the Bureau) provided new Intensity-Frequency-Duration (IFD) design rainfalls (http://www.bom.gov.au/water/designRainfalls/ifd/) in the form of point values as part of the upcoming revision of Australian Rainfall and Runoff. In design flood estimation however, catchments of significant size are often investigated and it is therefore necessary to provide areal design rainfall values. Since areal design rainfalls relate directly to point design rainfalls, a method commonly used is to apply a reduction factor to point values to obtain areal design rainfalls for a given catchment area, duration and annual exceedance probability (AEP). This factor is known as the areal reduction factor (ARF) and is the ratio between the areal and point design rainfalls for these variables.

The Bureau has developed ARFs for durations from 1 to 7 days from all of the data available in Australia (Podger *et al.* 2015) using a modified Bell's method (Bell, 1976) initially used with the application of the CRC-FORGE method (Siriwardena and Weinmann, 1996). As in the previous edition of Australian Rainfall and Runoff (ARR87) (Pilgrim 1987) recommended differing ARFs for differing zones, the potential for increased accuracy in long duration areal reduction factors by regionalising was investigated.

WMA Water has produced ARFs for durations from 30min to 1080min using a method more suitable to the data density and availability at those time-scales (Stensmyr and Babister, 2015). The values produced are directly applicable to the greater areas of Sydney, Melbourne and Brisbane, as areas outside of these did not have sufficient data to produce ARFs. As these short duration ARFs, generalised over the whole of Australia, will be used in conjunction with the long duration ARFs, it necessary to investigate the differences in data sets and to develop a strategy for defining ARFs between the two time-scales.

Section 2 describes the process and results from regionalising the long-duration ARFs; Section 3 highlights the differences in method and raw data for the long and short duration ARFs; Section 4 details the fitting of an equation to the short-duration ARFs; and Section 5 compares the modelled short and long duration ARFs and recommends the method for using them in conjunction.

2. REGIONALISING LONG DURATION AREAL REDUCTION FACTORS

Both the database for ARFs used for ARR87 (Pilgrim 1987) and the databases created in conjunction with the application of the CRC-FORGE method for various Australian states assumed changes in ARF relationships for different climatic regions or from state to state. By examining 2 test regions, the value of defining ARFs regionally was highlighted (Podger *et al*. 2015).

ARF regions were delineated using the basis of regions that were developed for the Bayesian Least-Squares Regression (BGLSR) (Figure 1) that was used to derive sub-daily statistics at daily rainfall gauges as part of the estimation of the new IFDs (Johnson *et al*. 2012). These regions were further modified based on data availability and meteorological factors including but not limited to seasonal rainfall zones (http://www.bom.gov.au/jsp/ncc/climate_averages/climate-classifications/index.jsp?maptype=seasb#maps), known topographical features, mean annual rainfall (MAR) and catchment boundaries. Initial region definitions that gave low accuracy ARF means were re-delineated to give the best possible results.

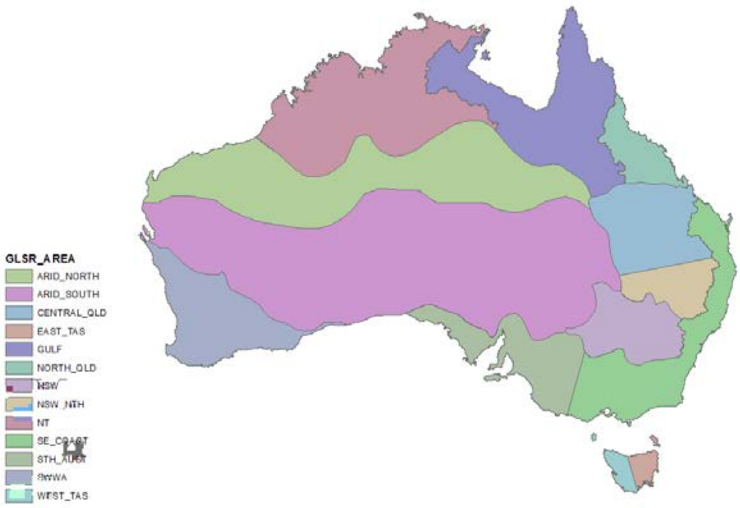

Figure 1 Regions used with the BGLSR

The method of deriving ARF relationships on a regional basis is identical to the method that was used to create the ARF relationship for all of Australia (Podger *et al*. 2015) except in the calculation of mean ARF values. Instead of using all possible values for a given catchment area, duration and AEP, samples were taken of hypothetical catchments whose centroids fell within the region of concern. Using the form of equation developed previously (Equation 1), the coefficients are determined via an evolutionary solving method and several statistical indicators were calculated to assess the performance of the model and the variability of data within the regional sub-set. For both Equations 1 and 2, the duration is in minutes, the area is in km^2 and AEP is a fraction of 1. For small areas that produce results greater than 1, the maximum value of 1 is used.

$$\begin{aligned}\mathbf{ARF_{AEP}} = \; & \mathbf{1 - a\left(Area^{b} - c.\log_{10}(duration)\right).duration^{-d}} \\ & \mathbf{+\, e.Area^{f}.duration^{g}\left(0.3 + \log_{10}(AEP)\right)} \\ & \mathbf{+\, h.10^{i.Area.\frac{duration}{1440}}\left(0.3 + \log_{10}(AEP)\right)} \qquad (1)\end{aligned}$$

Due to the fact that smaller sample sizes create less accurate estimates of sample means, R^2 values for regional estimates decreased and sample mean absolute errors (MAE) increased. Hence creating regions involved balancing between making areas that are most representative of ARFs in that area while also minimizing sampling errors. All statistical and visual tests indicated that using regions to define ARF relationships would create more accurate ARF predictions.

Using the above methods, regions were defined encompassing all of Australia resulting in the final set of regions shown in Figure 2. The equation used to define ARF values for all of Australia was fitted to the subsets of data areas associated with these regions to create the coefficients shown in Table 1. In the situation where a catchment lies across regional boundaries, it is recommended that the regional equation of the region in which most of the catchment lies be used.

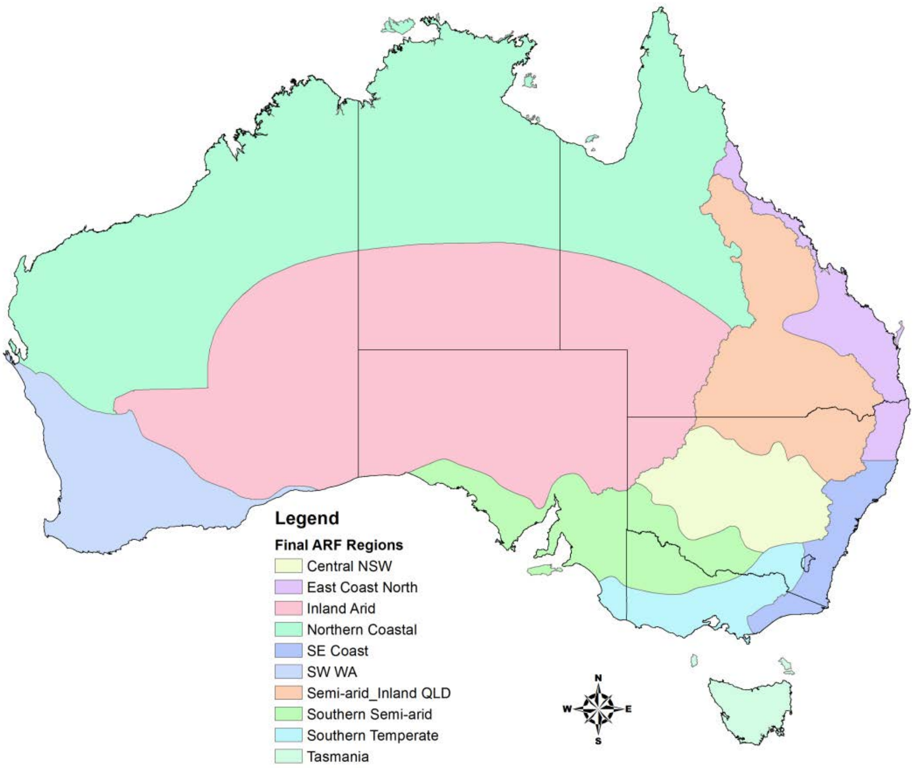

Figure 2 Final set of ARF regions

Table 1 Equation coefficients for the various regions

Region	a	b	c	d	e	f	g	h	i
East Coast North	0.327	0.241	0.448	0.36	0.00096	0.48	-0.21	0.012	-0.0013
Semi-arid Inland QLD	0.159	0.283	0.25	0.308	7.3E-07	1	0.039	0	0
Tasmania	0.0605	0.347	0.2	0.283	0.00076	0.347	0.0877	0.012	-0.00033
SW WA	0.183	0.259	0.271	0.33	3.845E-06	0.41	0.55	0.00817	-0.00045
Central NSW	0.265	0.241	0.505	0.321	0.00056	0.414	-0.021	0.015	-0.00033
SE Coast	0.06	0.361	0	0.317	0.0000811	0.651	0	0	0
Southern Semi-arid	0.254	0.247	0.403	0.351	0.0013	0.302	0.058	0	0
Southern Temperate	0.158	0.276	0.372	0.315	0.000141	0.41	0.15	0.01	-0.0027
Northern Coastal	0.326	0.223	0.442	0.323	0.0013	0.58	-0.374	0.013	-0.0015
Inland Arid	0.297	0.234	0.449	0.344	0.00142	0.216	0.129	0	0

3. COMPARING RAW LONG AND SHORT DURATION ARF DATA

WMA Water produced ARFs for durations of 30, 60, 120, 180, 360, 540, 720, 1080 minutes, AEPs of 50%; 20%, 10%, 5%, 2% and 1% and catchment areas of 10, 50, 100, 500 and 1000km^2. The method involved identifying all significant historical rainfall to create areal rainfall grids, calculating point and areal average rainfall for theoretical circular catchments, fitting a Generalised Extreme Value (GEV) distribution to the Annual Maximum Series (AMS) of point and areal rainfall and calculating ARF by dividing the areal rainfall quantiles by the point rainfall quantiles (Stensmyr and Babister, 2015). The locations that had sufficient data to derive these values were the greater areas around Sydney, Brisbane and Melbourne.

Given that these short-duration ARFs will be used in conjunction with the long-duration values and the method used for the long and short duration values are different, the two data sets were checked for substantial differences. Thus each capital city's sets of mean ARF were compared to the long-duration ARFs of the regions in which they resided (SE Coast, Southern Temperate and East Coast North).

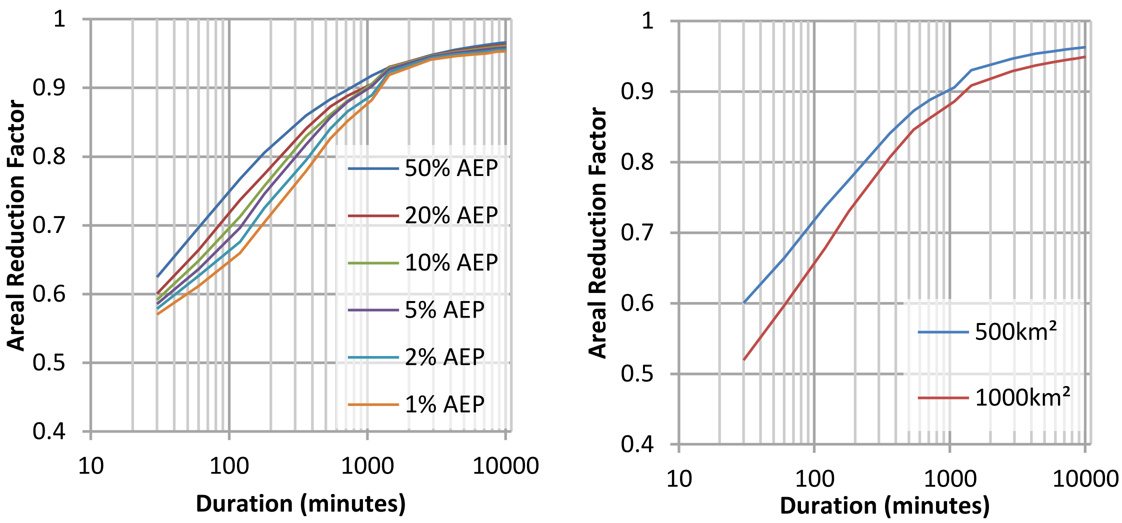

Figure 3 Lines through sub-daily and daily ARF for averaged Australian data for the 500km2 catchment area (left) and the AEP of 20% (right)

As can be seen in Figure 3, there are significant jumps in ARF between the two datasets for AEPs less frequent than 50%. Since the two datasets were created with different data and methods there are numerous possible reasons for the disparity. Possibly the most significant of these is that the short duration ARFs had to use shorter record lengths than the long duration ARFs as there are much more limited data available. The long duration ARFs used a minimum of 30 years AMS for the fitting of the GEV distributions whereas the short duration ARFs used a minimum of 3 years AMS. This means that

the disparity in accuracy for less frequent AEPs is considerable and comparing low AEP ARFs will be more an indication of bias in the GEV from using such a short AMS.

Another significant difference between the long and short duration ARFs is the number of sites that were available for use in the analysis. As can be seen in Table 2, there are vastly more daily read rain gauges available than continuous rainfall sites. This mean that the sample sizes of hypothetical catchments are much larger for the long duration ARFs and there is also likely to be more data located in all of the hypothetical catchments.

Table 2. Number of rain gauges within short and long duration regional areas

Region	Number of short duration sites	Number of long duration sites
Sydney	250	1642
Brisbane	166	1941
Melbourne	341	1665

Other possible influences on the differences are the calculation of areal rainfall. The long duration ARFs were derived using circular catchments and Thiessen polygons whereas the short duration values were derived using the Natural Neighbours algorithm and a polygon overlay algorithm as Thiessen polygons were shown to be unsuitable (Stensmyr and Babister, 2015). This means that the areal rainfall quantiles will be inherently different, as the short duration values consider rainfall outside of the catchment area.

Since there are much smaller samples in the short duration data, it was beneficial to define non-overlapping catchments multiple times to get more accurate estimates on the median ARF samples. This multiple sampling differs from the long duration method where the hypothetical catchments were only defined once and will affect the results.

Given that less frequent AEPs are the most affected by differences between the two datasets and are the values with the least accuracy, it will be beneficial to improve the accuracy of the short duration values. One of the most effective ways to decrease the uncertainty in ARFs is to increase the effective sample size, thereby increasing the accuracy of the mean ARF values. This was done by averaging ARF values for the regions of Sydney, Melbourne and Brisbane. While there are likely differences in ARF regionally, the increased uncertainty due to the reduced effective sample size will outweigh the potential benefits from using regional short duration ARFs, particularly for rarer AEPs.

4. APPLYING LONG-DURATION EQUATION TO SHORT DURATION ARFS

Since Equation 1 was shown to model the long-duration ARFs effectively, this form was also applied to the averaged short-duration values. The first term of the equation was found to achieve R^2 values that were reasonably close to those of the long-duration equations (Table 3), but the sections of the equations that model the changes of ARF with AEP were not as effective. This was also true for the data from the individual regions (before the averaging).

Table 3. Equation fit statistics for the 3 regions and their average

Region Options	$R^2_{0.5AEP}$	$R^2_{All\ AEP}$	MAE
Sydney	0.9945	0.9747	0.0173
Brisbane	0.9909	0.9622	0.0160
Melbourne	0.9970	0.9871	0.0089
Region mean values	0.9945	0.9769	0.0133

Given that there is reduced accuracy in the low AEP short duration ARFs, it is important to define changes of ARF with AEP to be consistent with the long duration values. If estimates were made that did not vary with AEP, then short duration ARFs would overlap the long duration values in certain regions at low AEPs.

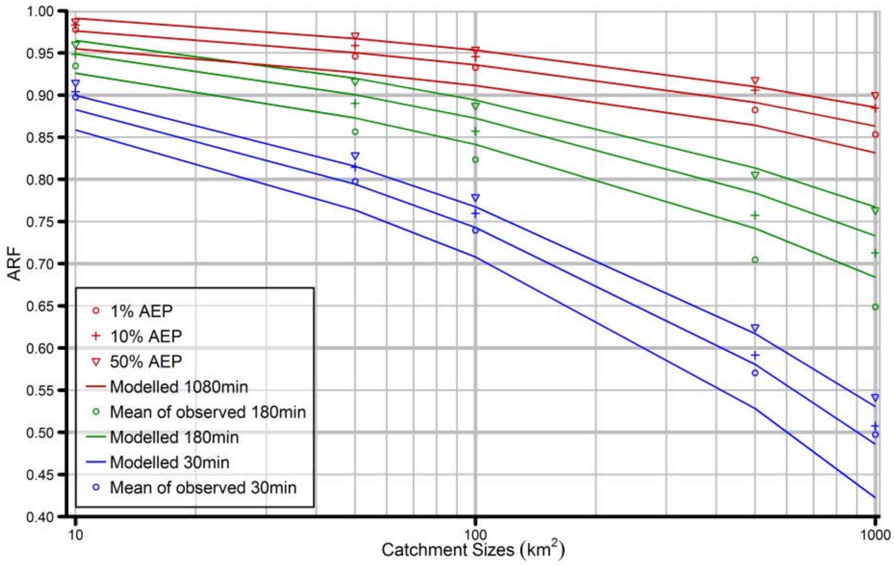

Figure 4 Comparison between 30, 180 and 1080 minute sub-daily ARFs that were averaged across all regions

The form of equation used assumes that in general the rate of change of ARF values in respect to AEP either increases or decreases with duration and/or catchment area. The short duration ARFs do not follow this assumption however, as there appears to be a maximum change of ARF with AEP around the 180 minute duration (Figure 4). This tendency is also present in all of the individual regions, however it is most pronounced in Sydney and Brisbane.

In order to adequately model the features that are unique to the short duration ARFs, the long-duration equation was modified so that the equation could capture the peak in ARF changes with AEP (Equation 2). The new form of equation achieved an improved R^2 value of 0.99233. As can be seen in Figure 5, the new form of equation better addresses the features that are unique to the short duration data and will therefore be adopted.

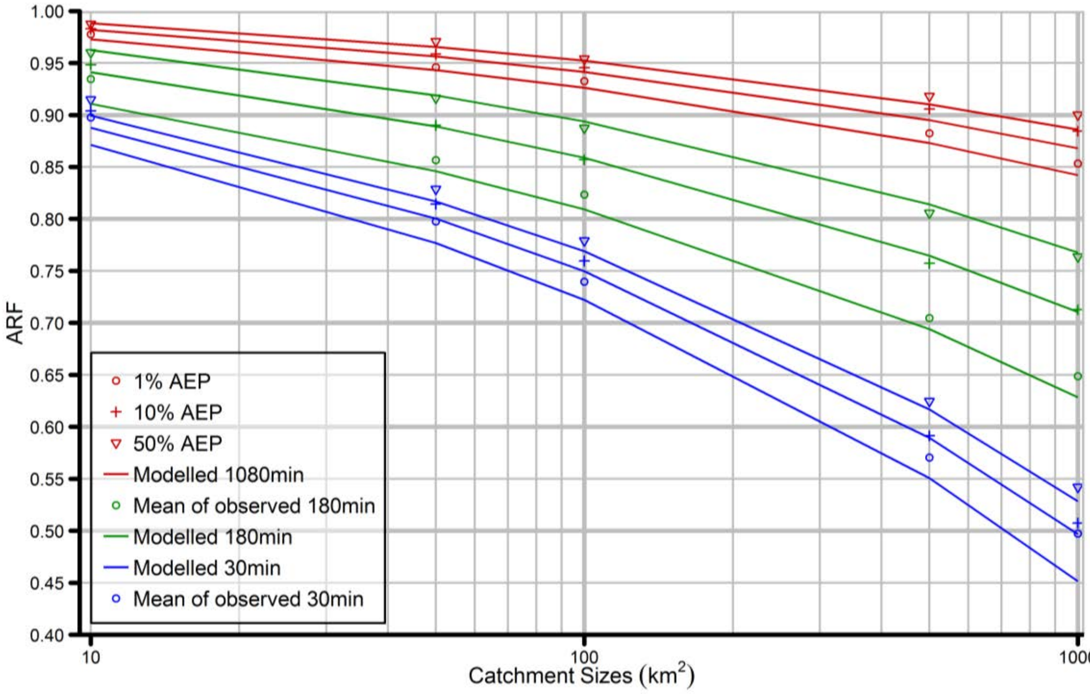

Figure 5 Comparison between 30, 180 and 1080 minute sub-daily ARFs that were averaged across all regions

$$\mathbf{ARF_{AEP}} = \mathbf{1} - \mathbf{0.287}\left(\mathbf{Area^{0.265}} - \mathbf{0.439} \times \mathbf{log_{10}(duration)}\right)\mathbf{.\,duration^{-0.36}}$$
$$+\mathbf{2.26} \times \mathbf{10^{-3}} \times \mathbf{Area^{0.226}.\,duration^{0.125}}(\mathbf{0.3} + \mathbf{log_{10}(AEP)})$$
$$+\mathbf{0.0141} \times \mathbf{Area^{0.213}} \times \mathbf{10}^{-\mathbf{0.021}\times\frac{(\mathbf{duration}-\mathbf{180})^2}{\mathbf{1440}}}(\mathbf{0.3} + \mathbf{log_{10}(AEP)}) \quad (2)$$

5. COMPARING MODELLED ARFS AND COMBINING RELATIONSHIPS

Short and long duration modelled ARFs were compared for all of the Australia averaged values (Figure 6), showing a generally good consistency between relationships. However, different ARF relationships were derived in each of the ten regions for defining long duration ARFs (see Figure 2) and it is essential that there is consistency between the short and long duration ARF equations to be adopted in each individual region.

To test the consistency of ARFs between the different data sets, the modelled short-duration ARFs for the 1080 and 720 minute durations were subtracted from the modelled long-duration ARFs for the 1080 minute and 1 day duration, producing the results in Table 4. The most notable part of these differences is the general trend for a gap between the long and short duration values and the overlap between the long duration 1440min and the short duration 720min ARFs. There is also a trend for the gap between ARFs to increase with catchment area and as AEP decreases.

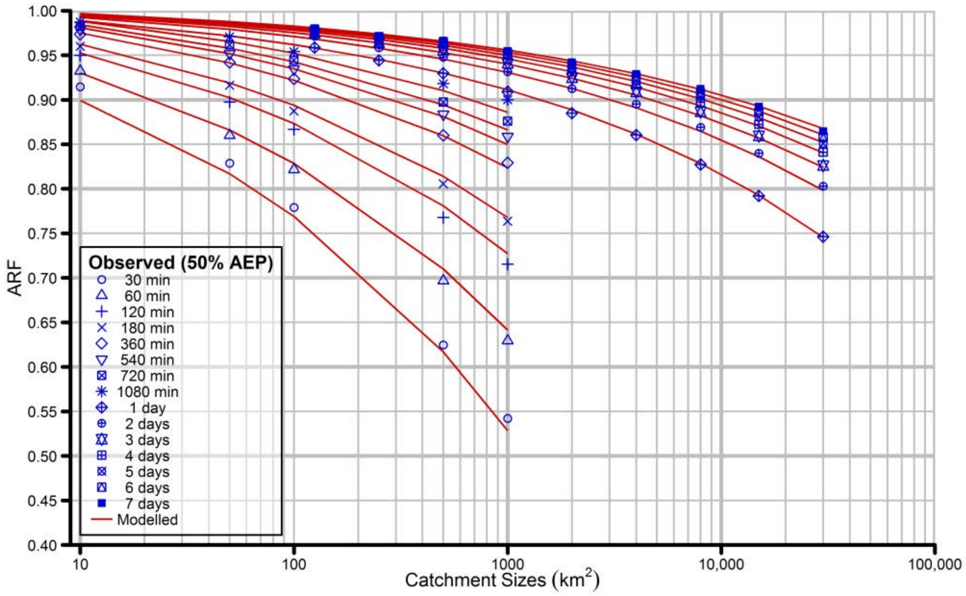

Figure 6 Areal reduction factors for all durations for all of Australia averaged values

Table 4 Differences between modelled ARF values for the short duration equation and the various daily regions

Long duration	Short duration	Minimum	1st quartile	Median	3rd quartile	Maximum
1440min	1080min	-0.0107	0.0056	0.0133	0.0261	0.0729
1080min	1080min	-0.0118	0.0017	0.0081	0.0187	0.0660
1440min	720min	-0.0075	0.0115	0.0228	0.0402	0.0907

As estimates may be required for ARFs that lie on the durational gap between the two datasets, it is important to have an effective way of interpolating between short and long-duration values. However straight interpolation will not work in the areas of overlap, so it is important to identify areas of the ARF relationship where the overlaps are occurring.

As can be seen in Table 5 if short duration values were used for the 1080min duration, this would result

in overlap over observed 100km^2 area catchments, which are derived from long duration ARF data, where there is more confidence in the results. Considering that using the short duration relationships up to the 720min duration would only create an overlap in catchment areas below 50km^2, where the long duration relationship is extrapolated and short duration data exists, not using the short duration relationships for durations greater than 720min was preferred.

Table 5 Minimum differences between modelled ARF for the short duration equation and all long duration equations for various catchment areas

Long duration	Short duration	10km^2	20km^2	50km^2	100km^2	200km^2	500km^2	1000km^2
1440min	1080min	-0.0107	-0.0057	-0.0033	-0.0012	0.0012	0.0047	0.0077
1440min	720min	-0.0075	-0.0010	0.0042	0.0084	0.0133	0.0209	0.0277

Since for the most part there is a reasonable gap between short duration 720min data and long duration 1440min data a linear interpolation between the long and short duration relationships was chosen for durations between 720min and 1440min. However where there is overlap between these datasets linear interpolation will violate ARF consistency, so the short duration 720min ARF will be used when the long duration ARFs are smaller than it since this only occurs in small catchment areas where the short duration relationship is more valid.

Since the forms of equations chosen is unsuitable for use below catchment areas of 10km^2, a linear interpolation from calculated ARFs at 10km^2 and a value of 1 at 0km^2 is recommended. This simple interpolation should be adequate considering the uncertainty in ARFs for small catchment areas and the very small changes in the values themselves. An example of a complete ARF set using these interpolation techniques can be seen in Figure 7.

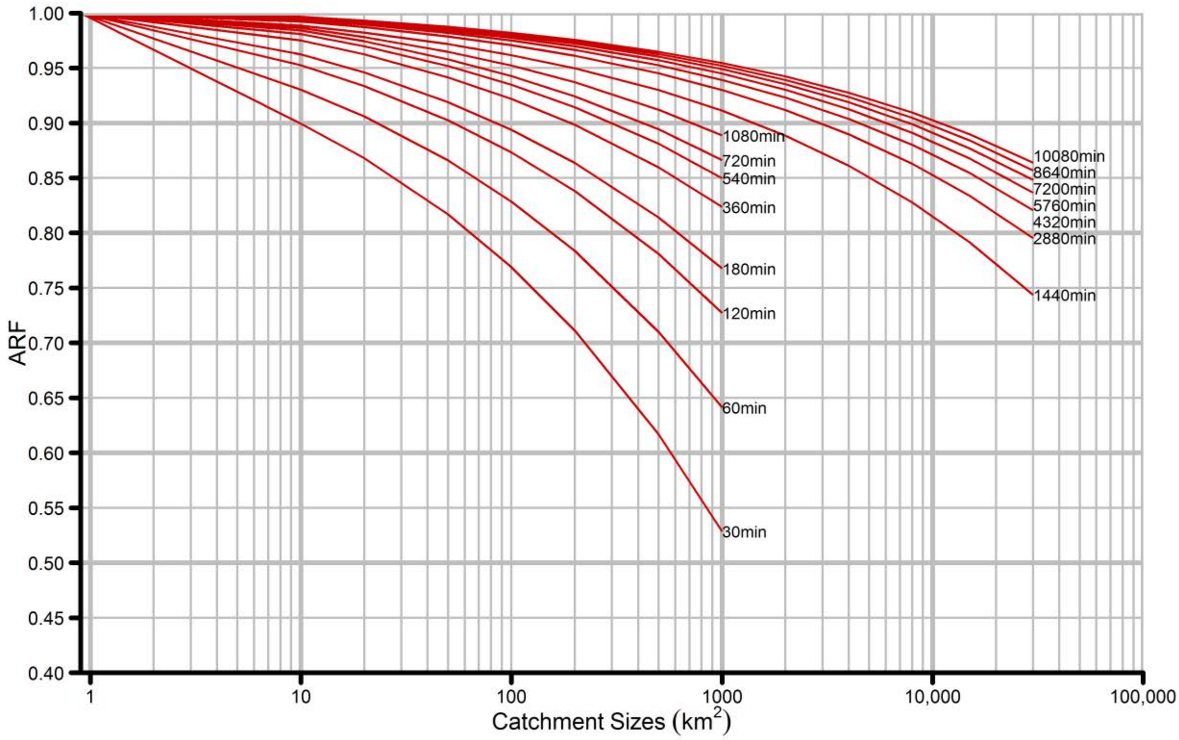

Figure 7 Modelled ARFs with the 1080min interpolation for the Southern Temperate regions and the 50% AEP

6. CONCLUSION

Trialling multiple long duration ARF regions highlighted that there is value in defining ARFs regionally. ARF regions were defined using the basis established in the BGLSR adopted for the new IFDs and taking into consideration various climatic characteristics, data availability and data consistency within a region. The form of equation used to define ARFs for all of Australia was used to define relationships for every region and the coefficients will be made available to users.

An investigation was carried out to determine potential issues in using the short duration ARFs produced by WMA Water in conjunction with the long duration ARFs created by the Bureau. This investigation highlighted many differences in data and method, but also demonstrated very similar ARF values for common catchment areas. Since the data were shown to be similar, the form of equation used for the long duration ARFs was applied to the short duration values, showing poor results. By modifying one term of the equation adopted for long duration ARFs an adequate fit was established and ARFs were defined for all durations from 30min to 720min, AEPs from 50% to 1% and catchment areas from $10km^2$ to $1000km^2$.

To define ARFs from durations between 720min and 1 day a simple linear interpolation was used, as it is effective, allows the use of short duration ARFs with long duration regional ARFs and is easily applied. When calculated long duration ARFs are smaller than 720min short duration ARFs, the short duration ARFs will be used as this only occurs in small catchment areas where the short duration ARFs are more valid. Linear interpolation is also recommended for defining ARFs at catchment areas less than $10km^2$, using the known value of ARFs being equal to 1 for $0km^2$ catchment areas.

7. REFERENCES

Bell, F.C., (1976), *The areal reduction factors in rainfall frequency estimation*. Natural Environmental Research Council (NERC), Report No. 35, Institute of Hydrology, Wallingford, U.K.

Johnson, F., Haddad, K., Rahman, A., Green, J., (2012), *Application of Bayesian GLSR to estimate sub daily rainfall parameters for the IFD Revision Project*. Presented at the Hydrology and Water Resources Symposium, Sydney, NSW, November 2012.

Pilgrim, DH, (ed)., (1987), *Australian Rainfall & Runoff – A Guide to Flood Estimation*, Institution of Engineers, Australia, Barton, ACT, 1987.

Podger, S., Green, J., Jolly, C., The, C., Beesley, C., (2015), *Creating Long Duration Areal Reduction Factors for the New Intensity-Frequency-Duration (IFD) Design Rainfalls*. Accepted for presentation at the Hydrology and Water Resources Symposium, Hobart, TAS, December 2015.

Siriwardena, L. and Weinmann, P. E., 1996*. Derivation of Areal Reduction Factors for Design Rainfalls in Victoria*. Cooperative Research Centre for Catchment Hydrology Working Document 99/1

Stensmyr, P., & Babister, M. (16 September 2015). Australian Rainfall and Runoff Revision Project 2: Spatial Patterns of Rainfall. Stage 3 Report*: Short Duration Areal Reduction Factors*. ARR Report.