Gridding of Design Rainfall Parameters for the IFD Revision Project for Australia

Cynthia The
Hydrologist, Climate and Water Division, Bureau of Meteorology, Sydney, Australia

Fiona Johnson
Hydrologist, Climate and Water Division, Bureau of Meteorology, Sydney, Australia

Michael Hutchinson
Professor of Spatial and Temporal Analysis, Fenner School of Environment and Society, Australian National University, Canberra, Australia

Janice Green
IFD Revision Project Manager, Climate and Water Division, Bureau of Meteorology, Canberra, Australia

As part of the revision of Australian Rainfall and Runoff, the Bureau of Meteorology has revised the Intensity-Frequency-Duration (IFD) design rainfall estimates. Daily point rainfall data are available for over 8000 sites across Australia. However, as IFD estimates are required Australia wide, the point data need to be gridded to provide estimates for areas where point data are not available. The grids serve as the basis for deriving IFD estimates for Annual Exceedance Probabilities (AEP) of 50% to 1%. Different gridding approaches are available to interpolate spatial data with thin plate smoothing splines being one widely used technique. This paper examines the gridding of the Generalised Extreme Value (GEV) parameters using a thin plate smoothing spline to estimate design rainfalls across Australia.

When gridding rainfall data, the interaction between topography and precipitation is important for the accurate interpolation of rainfall data. Large rainfalls tend to occur in areas of high elevation where the density of data is sparse as rainfall gauges are often located in more accessible lowland areas. As a consequence, at high elevations, rainfall is likely to be underestimated when it is spatially interpolated without reference to elevation. Thin plate smoothing splines allow for the incorporation of the topographic dependence of rainfall, which this study confirms as important for the modelling of Annual Maximum Series rainfall data. This investigation also explores the implications of Digital Elevation Model (DEM) resolution and the number of knot points when gridding precipitation using the smoothing spline software ANUSPLIN.

1. INTRODUCTION

As part of the revision of Australian Rainfall and Runoff, the Bureau of Meteorology has updated Intensity-Frequency-Duration (IFD) data. Daily point rainfall data, available for over 8000 sites across Australia has been gridded and serves as the basis for deriving IFD estimates for 50% to 1% Annual Exceedance Probability (AEP) events. The interaction between topography and precipitation is important for the accurate gridding of rainfall data. In areas of high elevation the density of data is sparse as rainfall gauges tend to be located in more accessible lowland areas. As a result at high elevations, rainfall is likely to be underestimated if the point data are interpolated without reference to elevation. The performance of the interpolation is known to be affected by a number of factors; including the digital elevation model (DEM) resolution, the number of data points, and the weighting and transformations applied to the data. This paper discusses these issues and recommends the best approaches for the IFD Revision Project. The following section provides a discussion of the approach used for the gridding of rainfall data for the project. This is followed by the results and a discussion of

their implications for the project. Finally conclusions are made for the approach adopted to produce the final IFD estimates.

2. METHOD

2.1. Thin plate smoothing splines

A number of methods are available to interpolate spatial point data. Thin plate smoothing splines and kriging are two of the most common methods used in gridding climate data (Hutchinson, 1995; Jeffrey et al., 2001; Sharples et al., 2005; Beesley et al., 2009; Hutchinson et al., 2009). Thin plate splines have been chosen as the analysis method for the IFD Revision Project due to their ability to model the spatially coherent signal in the rainfall data as well as removing the noise inherent in the point data (Hutchison and Gessler, 1994). Thin plate splines are readily accessed through the use of the ANUSPLIN Version 4.37 software (Hutchinson, 2007). A beta release of ANUSPLIN Version 4.4 was used for this study. The beta version has the capability to provide individual cross validated estimates of the fitted spline surface as well as an ability to use error variances to weight the contribution of the transformed target variable. The software has also been simplified by combining the selection of the initial knot set with the surface fitting procedure in a single program.

The general model for the trivariate thin plate spline is shown in Equation (1)

$$z_i = f(x_i) + e_i \quad for\, i = 1,...,N \tag{1}$$

where z_i are the observed data values for N stations, x_i is a 3-dimensional vector of spline independent variables, consisting of longitude, latitude and appropriately scaled elevation, and f is an unknown smooth function of x_i. The error term e_i is discussed further in Section 3.3.

The aim of fitting the thin plate spline to the data is to estimate the function f by minimising a penalised residual sum of squares, where the penalty is a trade off between the residual sum of squares and the roughness of the spline function, as determined by a smoothing parameter. If the smoothing parameter approaches zero then the spline fits the data points exactly. As the value of the smoothing parameter increases, a second order spline, the usual default, approaches a linear least squares regression fit (Wahba 1990). The smoothing parameter is normally optimised by minimising the Generalised Cross Validation, as discussed in Section 2.3.

2.2. Rainfall parameters for analysis

Previous investigations for the IFD revision project have shown that the Generalised Extreme Value (GEV) distribution provides a good fit to the Annual Maximum Series data from individual sites and also from regionalised relationships. The GEV has been fitted to regionalised rainfall as reported in Johnson et al. (2012), using a region of influence approach. This approach involves scaling the regionalised GEV growth curve by the at site mean rainfall, which is also called the “index rainfall”.

The GEV parameters are related to the rainfall quantiles according to the following equation:

$$q(F) = \xi + \alpha\left\{1 - \left(-\log(F)\right)^{\kappa}\right\}/\,\kappa \qquad k \neq 0 \tag{2}$$

where ξ, α and κ are the location, scale and shape parameters, and F is the quantile of interest.

There are two alternatives for providing the final rainfall quantile estimates at gridded locations across Australia. The first is to grid the three parameters that describe the GEV distribution at each station location (i.e. the index rainfall and the shape and scale parameter). Alternatively the rainfall quantiles of interest can be calculated at each station location and used as the inputs to the ANUSPLIN gridding. Earlier investigations have shown that there is little practical difference in the estimates from

the two approaches for the final gridded estimates of the 50% to 1% AEP rainfall quantiles. Thus the ANUSPLIN analysis reported here discusses gridding of the GEV parameters due to the flexibility of this approach to estimate any desired rainfall quantile. The gridding optimisation reported here has only been carried out for the index rainfall as there is a better conceptual understanding of the expected output for this parameter because the relationship between the rainfall and topography is clearer. In addition, there is a large range in the values of index rainfalls across Australia which makes it easier to isolate inconsistencies in the gridded data. The results reported here are for the 1 day to 7 day duration rainfall events. The data used for the analysis is a set of 8075 Bureau of Meteorology owned daily rainfall stations with more than 30 years of data. The station locations are shown in Figure 1.

2.3. Model optimisation and evaluation

The optimisation of the thin plate spline fits and the evaluation of the different modelling strategies have been made through the use of several summary statistics as described below:

- Generalised Cross Validation (GCV) of each fitted spline surface. The GCV is calculated by implicitly removing each data point in turn and calculating the residual of the surface fitted using all other data points for a fixed value of the smoothing parameter. The GCV is a weighted sum of squares of these residuals. The amount of data smoothing imposed by the thin plate spline is determined by minimising the GCV with respect to the value of the smoothing parameter. The resulting minimum GCV for a particular spline model option can then be compared with the GCV for other models to help determine the best model.
- Cross Validation Statistics – each data point can be implicitly left out of the analysis to calculate the individual residual from the fitted surface without that station. The Mean Absolute Error (MAE) and Root Mean Square Error (RMSE) of these individual unweighted residuals can also be used to evaluate the overall predictive error of the fitted spline surface.
- Cross validation statistics on a high quality data subset – the cross validation statistics are also reported for a spatially representative set of 500 stations across Australia, all of which have at least 50 years of data (Figure 1). The high quality station subset was selected using SELNOT, one of the supporting programs for ANUSPLIN which attempts to sample equally in the independent spline variable space (Hutchinson, 2007). This spatially representative sample reduces the bias in the cross validation statistic that can arise from data sets with uneven spatial density (Hutchinson et al. 2009), as is evident in the data plotted in Figure 1.

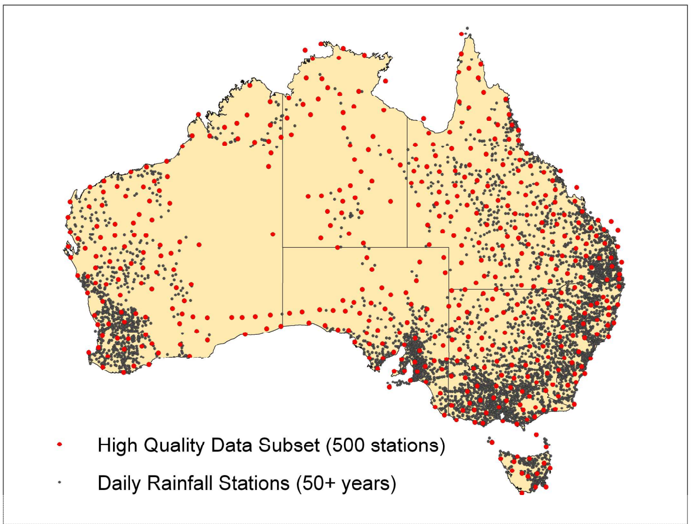

Figure 1 Location of high quality data subset used in cross validation.

Given the skewness of the distributions of index rainfalls across Australia, it is considered that the MAE is a more reliable validation statistic as the RMSE may be biased by poor fits to sites with large rainfalls. The main statistic used in the validations is therefore the MAE of the spatially representative

high quality data subset. The signal of the fitted thin plate spline is also reported in the results presented in Section 3. The signal measures the effective number of parameters used in the fitted spline. It can be used to assess whether the fitted spline is appropriate. If the signal is too large then this can indicate that there are insufficient data points or that errors in the data are correlated. Hutchinson and Gessler (1994) suggest that the signal should not exceed about the half the number of knots.

The following questions are addressed in this investigation:

- Optimum grid resolution
- Optimum number of knots to be used for the thin plate smoothing spline
- Impact of weighting the inputs using their error variances
- Appropriate transformations for the index rainfall
- Impact of including elevation in the thin plate smoothing spline as a predictor of index rainfall

For each of these, a natural logarithm transform of the index rainfall has been used. This choice of transformation is further explored in Section 3.4. Analysis has been carried out for rainfall event durations from 1 day to 7 days. Each spline surface is fitted independently and the ANUSPLIN log file reports results for each duration's surface separately. Summary statistics averaged across all durations are reported below.

3. RESULTS AND DISCUSSION

3.1. Optimum Grid Resolution

The grid resolution is used in fitting the thin plate smoothing splines through the use of area averaged elevations at the station locations rather than the point station elevations. This is considered to be desirable for several reasons. Firstly the recorded station point locations may not be accurate for rainfall stations that have not been recently open and station elevation information may not be available for all rainfall stations. Secondly it is thought that by using the elevations from an area averaged DEM, the thin plate smoothing splines will be using information at the scales that rainfall processes generally operate i.e. in the order of a few km (Sharples et al., 2005).

Grid resolutions from 0.0025 to 0.07 degrees (approximately 0.25 to 7 km) were tested for the surfaces of index rainfall. For the initial testing 2000 knots were used for each thin plate smoothing spline. Summary results averaged over all durations are presented in Table 1. The minimum predictive error for the high quality data set (far right column in Table 1) is obtained from the 0.025 degree (approximately 2.5 km) resolution. Results are fairly similar for the five finest grid resolutions.

Table 1 Summary statistics for grid resolution optimisation using 2000 knots

2000 knots	All Stations		Cross Validated Transformed Variable		Cross Validated Untransformed Variable		Cross Validated High Quality Stations	
Resolution	**Signal**	**GCV**	**RMSE**	**MAE**	**RMSE**	**MAE**	**RMSE**	**MAE**
0.0025	1440.8	0.0853	0.0871	0.0628	14.30	7.31	18.65	8.31
0.010	1431.2	0.0843	0.0861	0.0622	14.20	7.25	18.19	8.20
0.020	1417.3	0.0842	0.0856	0.0619	14.00	7.20	17.80	8.22
0.025	1455.3	0.0823	0.0840	0.0611	13.70	7.07	17.70	8.12
0.030	1453.0	0.0828	0.0844	0.0612	13.50	7.04	18.38	8.27
0.050	1449.9	0.0827	0.0842	0.0609	13.40	6.98	18.15	8.21
0.070	1415.0	0.0857	0.0865	0.0627	13.70	7.23	18.37	8.51

ANUSPLIN reports the largest residuals between the fitted and point estimates. These residuals were checked for obvious data errors, that their location was correct and that the period of record was representative. This was found to be the case for all stations, as quality control and previous

ANUSPLIN analyses had identified problem stations. To reduce the deviations of the fitted spline surface from the point estimates, an additional 20 knots, obtained from the 20 largest residuals that were not already knots, were used in the second round of DEM resolution testing. This was based on adding 1% to the initial number of knots. The results from this work are shown in Table 2. The optimum resolution appears to be the 0.025 degree DEM resolution according to the MAE for the high quality data set followed closely by the 0.05 degree DEM. Thus the 0.025 degree and 0.05 degree DEM resolutions will be used for the other optimisation tests.

Table 2 Summary statistics for grid resolution optimisation using 2020 knots

2020 knots	All Stations		Cross Validated Transformed Variable		Cross Validated Untransformed Variable		Cross Validated High Quality Stations	
Resolution	Signal	GCV	RMSE	MAE	RMSE	MAE	RMSE	MAE
0.0025	1521.9	0.0829	0.0859	0.0623	14.00	7.23	18.82	8.26
0.010	1509.1	0.0821	0.0852	0.0618	14.00	7.19	18.39	8.24
0.020	1489.5	0.0817	0.0843	0.0614	13.60	7.08	18.44	8.30
0.025	1531.1	0.0804	0.0835	0.0610	13.10	6.97	17.76	8.05
0.030	1530.3	0.0806	0.0834	0.0609	13.30	6.99	18.41	8.21
0.050	1544.5	0.0794	0.0823	0.0602	13.00	6.86	17.31	8.11
0.070	1496.9	0.0820	0.0841	0.0613	13.30	7.05	17.77	8.35

3.2. Optimum number of knots

The results in the previous section show that the choice of DEM resolution is strongly affected by the number of knots used in the thin plate smoothing spline. Previous investigations for the IFD revision project, not reported here, have shown that the number of knots should be around 2000 and 3000 to provide appropriate definition of the spatial variations in index rainfall when using around 8000 rainfall stations. It was found that the complexity of the fitted spline did not appreciably increase with larger knot sets.

Tests for the 0.025 degree and 0.05 degree resolutions were carried out with 2000 knots and 2500 knots. For both knot sizes, the number of knots was increased by up to 2% in two stages (e.g. initially 2000 knots, then 2020 knots and 2040 knots). Results clearly indicate the addition of knots improves the performance of the model resulting in lower cross validation error statistics (Table 3) for all stations and the set of high quality stations. Based on these statistics, it is recommended that 2550 knots are used for both DEM resolutions.

Table 3 Summary statistics for optimising number of knots

DEM Resolution	All Stations		Cross Validated Transformed Variable		Cross Validated Untransformed Variable		Cross Validated High Quality Stations	
No. of knots	Signal	GCV	RMSE	MAE	RMSE	MAE	RMSE	MAE
0.025 degree DEM								
2000	1455.3	0.0823	0.0840	0.0611	13.70	7.07	17.70	8.12
2020 - +1%	1531.1	0.0804	0.0835	0.0610	13.10	6.97	17.76	8.05
2040 - +1%	1575.4	0.0794	0.0834	0.0609	13.10	6.94	17.95	8.03
2500	1694.4	0.0810	0.0827	0.0600	13.50	6.93	17.63	8.06
2525 - +1%	1804.7	0.0788	0.0819	0.0598	12.90	6.83	17.61	8.00
2550 - +1%	1886.2	0.0771	0.0812	0.0594	12.80	6.76	17.73	8.02
0.05 degree DEM								
2000	1449.9	0.0827	0.0842	0.0609	13.40	6.98	18.15	8.21
2020 - +1%	1544.5	0.0794	0.0823	0.0602	13.00	6.86	17.31	8.11
2040 - +1%	1591.2	0.0782	0.0820	0.0601	12.80	6.82	17.79	8.12
2500	1706.0	0.0799	0.0818	0.0595	13.20	6.83	17.83	8.07
2525 - +1%	1824.5	0.0772	0.0805	0.0589	12.70	6.70	16.71	7.81
2550 - +1%	1896.5	0.0759	0.0801	0.0588	12.50	6.67	16.91	7.88

3.3. Impact of weighting the inputs using their error variances

The thin plate spline uses a model for the errors at individual sites as shown in Equation 3.

$$e_i \sim N\left(0, w_i \sigma^2\right) \tag{3}$$

Where N denotes a normal distribution, w_i is the relative error variance at station i and σ^2 is the error variance term which is constant across all points.

In the models reported in Section 3.1 and 3.2, the errors were assumed to be constant across all stations (i.e. w_i all equal to one). However this neglects the variance information that is available for each point estimate of the rainfall parameters. For the index rainfall, the point estimates of the error variance were estimated as the square of the standard error of the index rainfall estimates.

Table 4 compares the cross validation error estimates for the unweighted cases (as reported in Table 3) with the corresponding case where the inputs are weighted by their error variances. The results are reported for the 2550 knot case since this was shown to provide the best results in Table 3. Since the standard errors are inversely proportional to the record length, using the error variances in ANUSPLIN allows the longer record lengths stations to be given more weight in the fitted spline. In addition ANUSPLIN is given information on the variability of the rainfall at the location. The weighted analyses have smaller signals, indicating more robust analyses. However, the differences between the weighted and unweighted estimates are quite small. A review of the resulting surfaces shows that the largest differences occur in data sparse areas. The MAE for the high quality data set is reduced by using the error variance weighting although some of the other summary statistics are increased slightly using the differential weighting.

Table 4 Summary statistics for the impact of weighting with error variances

2550 knots	All Stations		Cross Validated Transformed Variable		Cross Validated Untransformed Variable		Cross Validated High Quality Stations	
Case	**Signal**	**GCV**	**RMSE**	**MAE**	**RMSE**	**MAE**	**RMSE**	**MAE**
0.025 deg. unweighted	1886.2	0.0771	0.0812	0.0594	12.80	6.76	17.73	8.02
0.025 deg. weighted	1725.6	0.0854	0.0812	0.0591	13.10	6.80	17.95	7.98
0.05 deg. unweighted	1896.5	0.0759	0.0801	0.0588	12.50	6.67	16.91	7.88
0.05 deg. weighted	1753.5	0.0835	0.0795	0.0581	12.70	6.65	17.15	7.84

3.4. Appropriate transformations for index rainfall

ANUSPLIN offers a choice of transformations for the dependent variable. A transformation may be required if the variable to be predicted is strongly skewed. In these cases without the transformation, the results are likely to be biased to the high values. Two alternatives are a square root transformation and a natural logarithm transformation. ANUSPLIN transforms the error variances according to the chosen data transformation and the error analysis for the fitted surface can be reported in both transformed and untransformed variable space. ANUSPLIN also corrects for the bias that is induced in the back-transformed fitted spline values (Neyman and Scott, 1960). The fitted thin plate smoothing splines were tested using all transformation options.

Results are based on tests using 2500 knots for simplicity and using the error variances to weight the inputs to the analysis. The log transformation clearly provides the best performance for all cross validation statistics and also minimises the signal of the resulting surface (Table 5). The latter gives rise to a more robust analysis and also enables a more effective analysis for a given limited number of knots. As it was very clear that the log transformations provided the best results, it was only tested on the 0.05 degree DEM.

Table 5 Summary statistics for applying different transformations to index rainfall

Resolution and transformation applied	All Stations		Cross Validated Transformed Variable		Cross Validated Untransformed Variable		Cross Validated High Quality Stations	
Case	**Signal**	**GCV**	**RMSE**	**MAE**	**RMSE**	**MAE**	**RMSE**	**MAE**
0.05 deg no trans	2052.8	12.2000	-	-	14.90	7.35	19.98	8.85
0.05 deg sqrt trans	1797.3	0.4760	0.4950	0.3140	14.20	7.08	19.06	8.39
0.05 deg log trans	1592.5	0.0881	0.0818	0.0592	13.40	6.85	18.11	8.08

3.5. Impact of elevation as a predictor of index rainfall

Elevation is considered to play an important role in the accuracy of gridded rainfall data because of the topographic dependence of rainfall (Hutchinson, 1998; Sharples et al., 2005). Past studies have shown that position and vertically exaggerated elevation are useful independent variables to use in interpolating mean rainfall data (Hutchinson, 1995). The effect of vertically exaggerated elevation on rainfall has been demonstrated by showing that the root mean square residuals of withheld data displayed a distinct minimum when station elevations were exaggerated by a factor of 100 (Hutchinson, 1998;1995). The appropriate relative scaling for these analyses was achieved by using longitude and latitude in units of decimal degrees and elevations in km.

To assess the importance of elevation, tests were conducted by fitting splines with error variance weighted data that excluded elevation and used only longitude and latitude. The results were compared to the earlier results for the 0.025 and 0.05 degree DEM based on 2500 knots. Results indicate that the inclusion of elevation as a predictor of index rainfall leads to a significant reduction in the cross validation error statistics by as much as 10% (Table 6).

Table 6 Summary statistics for the impact of elevation

2500 knots	All Stations		Cross Validated Transformed Variable		Cross Validated Untransformed Variable		Cross Validated High Quality Stations	
Case	Signal	GCV	RMSE	MAE	RMSE	MAE	RMSE	MAE
Elevation excluded	1426.6	0.11	0.0935	0.0661	14.80	7.61	21.95	8.86
0.025deg DEM	1566.1	0.0898	0.0829	0.0599	13.80	6.98	18.10	8.11
0.05 deg DEM	1592.5	0.0881	0.0818	0.0592	13.40	6.85	18.11	8.08

4. CONCLUSIONS

The investigations in ANUSPLIN reported in this paper have been used to recommend a strategy to provide grids of rainfall parameters for the IFD Revision Project. Although the 0.025 degree resolution performed the best in most cases, there is marginal difference when compared the 0.05 degree DEM results. For the purposes of the IFD Revision Project it is recommended that the 0.025 degree DEM resolution should be adopted because of the additional definition it can provide for the final IFD grids in urban areas where practitioners are likely to need IFDs derived for small catchments.

The choice of resolution is also influenced by the number of knots used. The optimum number of knots was found to be 2550, because it led to the lowest cross validated MAE for the spatially representative High Quality stations. Variance weighting was shown to provide more robust results in data sparse areas. However the use of variance weighting for the IFD revision project is still being considered due to possible difficulties in calculating the variances for sub-daily durations. This will be an area for future research. The natural log transformation for index rainfall provides the best performance for all validation statistics. Elevation has been confirmed as an important covariate, improving the performance of thin plate splines and will be incorporated in all models along with longitude and latitude. Figure 2 shows the final index rainfall map for the 1 day rainfall event, based on the model parameters recommended from this investigation.

The spatial pattern in Figure 2 is reasonably similar to the pattern of Mean Annual Rainfall (MAR), but with simpler gradients over central Australia and possibly a more consistent coastal gradient along the east coast of Australia. The latter reflects the consistent availability of atmospheric moisture along the east coast. The area of high MAR in south western Tasmania is not reflected in the index rainfall. Further investigations are required to assess whether this is a result of the daily station density in this region or is caused by nature of the meteorological conditions affecting this region. Other fine scale features in this map largely reflect local variations in topography, but also occasional data outliers in some data sparse areas.

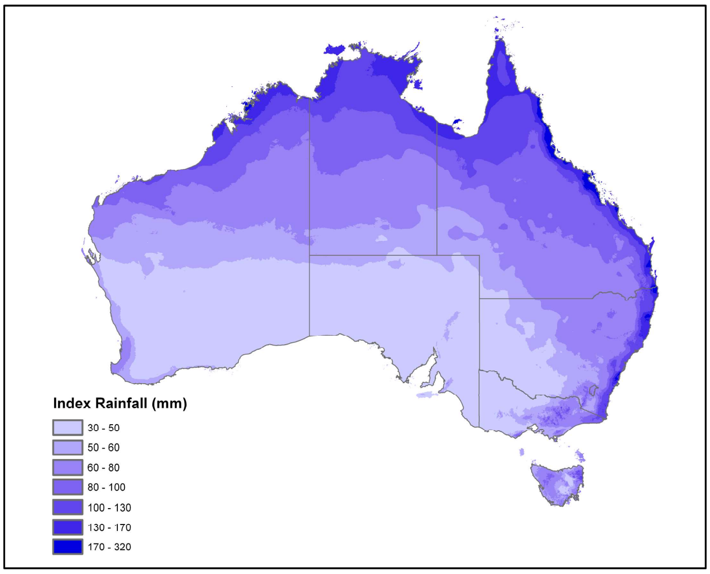

Figure 2 1 day index rainfall map (0.025 degree DEM, 2550 knots)

5. REFERENCES

Beesley, C.A., Frost, A.J and Zajaczkowski, J. (2009), A comparison of the BAWAP and SILO spatially interpolated daily rainfall datasets, In*: 18th World IMACS Congress and MODSIM09 International Congress on Modelling and Simulation. Modelling and Simulation Society of Australia and New Zealand, International Association for Mathematics and Computers in Simulation*, July 2009, Cairns, Australia.

Hutchinson, M.F and Gessler, P.E. (1994), *Splines – more than just a smooth interpolator*, Geoderma 62: 45-67.

Hutchinson, M. F. (1995), *Interpolating mean rainfall using thin plate smoothing splines,* International Journal of Geographical Information Systems 9(4): 385-403.

Hutchinson, M.F. (1998), *Interpolation of rainfall data with thin plate smoothing splines – part 2: Analysis of topographic dependence*, Journal of Geographic Information and Decision Analysis 2(2): 152-167.

Hutchinson, M.F. (2007), *ANUSPLIN Version 4.37 User Guide*, The Australian National University. Centre for Resources and Environmental Studies, Canberra, Australia.

Hutchinson, M.F., McKenney, D.W., Lawrence, K., Pedlar, J.H., Hopkinson, R.F., Milewska, E. and Papadopol, P. (2009) *Development and testing of Canada wide interpolated spatial models of daily minimum-maximum temperature and precipitation for 1961-2003*, Bull. Amer. Meteor. Soc 48: 725 -741.

Jeffery, S.J., Carter, J.O, Moodie, K.B, and Beswick, A.R. (2001), *Using spatial interpolation to construct a comprehensive archive of Australian climate data*, Environmental Modelling & Software 16: 309-330.

Johnson, F., Xuereb, K., Jeremiah, E. and Green, J. (2012), *Regionalisation of rainfall statistics for the IFD Revision Project*, Hydrology and Water Resources Symposium 2012, Sydney, Australia.

Neyman, J. and Scott, E.L. (1960). *Correction for bias introduced by a transformation of variables.* The Annals of Mathematical Statistics 31(3): 643-655.

Sharples, J.J., Hutchinson M.F. and Jellett D.R. (2005), *On the horizontal scale of elevation dependence of Australian Monthly Precipitation,* Journal of Applied Meteorology 44: 1850-1865.

Wahba, G. (1990), *Spline models for Observational Data*, CBMS-NSF Regional Conference Series in Applied Mathematics, Vol. 59, Society for Industrial and Applied Mathematics, 169 pp.